The economics of orbital transportation

By Akhil Rao*

Transportation through outer space involves a sequence of transfer orbits to move a payload from origin to destination. Transfer orbits are often chosen with the goal of minimizing either the energy required or time taken for the trip. Commercial shippers will instead choose transfer orbits to maximize delivery profits. When payment is received upon delivery and the opportunity cost of funds is positive, profit-maximizing transfer orbits may minimize neither trip energy nor time. Such "interior transfer orbits" balance the marginal present value from quicker delivery against the marginal cost of energy expenditures to reduce delivery time.

The current growth in commercial space activity promises and is predicated on a space economy where payloads will be transported through outer space. Such transportation is likely to remain in Earth's orbital space for the immediate future, but may soon expand to include the Earth-Moon system and beyond. Transportation activities to be conducted in Earth's orbital space include satellite servicing, while activities in the Earth-Moon system and beyond include space mining operations and the construction and maintenance of space habitats.

Transporting objects through outer space involves calculating "transfer orbits" which link circular orbits at the origin and destination. Even when the origin is a point on a planetary surface or in deep space, the path from origin to destination may be modeled as a sequence of transfer orbits. Optimizing transfer orbits is critical for mission success. The majority of existing scientific and engineering research on transfer orbits focuses on paths which, subject to idiosyncratic mission constraints, minimize either the energy required (e.g. Hohmann and bi-elliptical orbits) for the trip or the time taken (e.g. fast orbits). Minimizing trip energy or trip time is typically well-suited for budget-constrained research missions or missions delivering time-sensitive payloads.

Commercial shipping through outer space involves a different objective: maximizing profits. The shipper is paid for their services at least partially upon delivery, and discounts the future according to their opportunity cost of waiting for payment. Making the delivery costs energy, and within technological limits additional energy can be used to reduce the time taken. Feasible delivery times can range from the order of hours or days for delivery in the Earth-Moon system

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to the order of months or years for deeper-space missions. The shipper's objective therefore incorporates the time value of the payment received upon delivery and the current costs incurred to supply energy for the trip.

Solving the commercial shipper's profit maximization problem reveals that energy-minimizing transfer orbits are only optimal when the payment upon delivery and the opportunity cost of waiting for payment are sufficiently low relative to the marginal energy cost of additional delivery speed. Conversely, time-minimizing transfer orbits are only optimal when the payment upon delivery and the opportunity cost of waiting for payment are sufficiently high. For intermediate cases, the profit-maximizing sequence of transfer orbits is "interior": it minimizes neither the trip energy nor the trip time. Instead, it balances the marginal present value from receiving the payment sooner against the marginal cost of the energy required to reduce delivery time.

I. A model of commercial orbital transportation

A commercial space shipper transports a payload from an origin to a destination at non-relativistic speeds. The payment received upon delivery is p, discounted exponentially at rate r.[1] The shipper uses J units of energy to transport the payload in T units of time. Given a level of energy, the minimum-time path from origin to destination takes $\bar{T} > 0$ units of time, while given a delivery time the minimum-energy path takes $\bar{J} > 0$ units of energy. Energy for the trip costs the shipper $c(J)$ units of money, where costs are increasing and weakly convex ($c' > 0, c'' \geq 0$). Shipments may be delivered using a transport technology such as rockets, ion thrusters, or light sails. The transport technology and environmental conditions such as the relative masses and positions of celestial bodies along the path determine the feasible transformations between energy and time. Abstracting from the choice of transport technology and the present environmental conditions, the shipper requires $f(T)$ units of energy to achieve delivery time T. Additional delivery speed requires an increasing amount of energy ($f' < 0$). Instantaneous and zero-energy deliveries are infeasible ($\lim_{T\to 0} f(T) = \infty, \lim_{T\to\infty} f(T) = 0$), so the minimum-time path costs energy ($J(\bar{T}) > 0$) and the minimum-energy path takes time ($T(\bar{J}) > 0$).[2] Minimum-time and minimum-energy paths differ, so $\bar{J} \neq J(\bar{T})$ and vice versa. The shipper chooses the energy supply for the trip J and delivery time T to maximize profits, solving

[1]A flat up-front payment can be ignored without loss of generality, since it enters delivery profits as an additive constant. Up-front payments which vary with delivery time can alter marginal trade-offs, but as long as customers pay more for earlier delivery the essence of the main result is unchanged.

[2]This rules out degenerate deliveries where origin is the same as destination.

$$\begin{aligned} \max_{T,J} \quad & e^{-rT}p - c(J) \\ \text{s.t.} \quad & f(T) = J, \\ & J \geq \bar{J},\ T \geq \bar{T}. \end{aligned} \tag{1}$$

Program 1 describes the choice of profit-maximizing shipping energy and delivery time for any type of trajectory from origin to destination, whether a simple single-impulse transfer orbit or a more complex sequence of maneuvers. While simple deliveries across relatively short distances will likely have a single profit-maximizing solution, more complex deliveries across longer distances may have multiple solutions. The properties of the energy supply, technology, and environmental conditions encoded in c and f will determine the uniqueness of the profit-maximizing orbital transportation solution.

Define the marginal cost of energy to reduce delivery time ("marginal cost of quicker delivery") in terms of money per unit time as $-c'(J)f'(T)$ and the marginal present value from receiving payment sooner ("marginal benefit of quicker delivery") as rpe^{-rT}.[3] Proposition 1 describes when it is profit-maximizing to use a minimum-energy orbit, a minimum-time orbit, or an interior orbit.

Proposition 1 (Optimal transfer orbits)**.** *The profit-maximizing sequence of transfer orbits minimizes trip energy ($J = \bar{J}, T > \bar{T}$) when*

$$rpe^{-rT} \leq -c'(\bar{J})f'(T), \tag{2}$$

minimizes trip time ($J > \bar{J}, T = \bar{T}$) when

$$rpe^{-r\bar{T}} \geq -c'(J)f'(\bar{T}), \tag{3}$$

and minimizes neither trip energy nor trip time when there exist $T > \bar{T}$ and $J > \bar{J}$ such that

$$rpe^{-rT} = -c'(J)f'(T). \tag{4}$$

The proof is straightforward but cumbersome, so is relegated to the appendix.

Proposition 1 shows that shippers receiving no payment upon delivery will find it optimal to minimize trip energy. Conversely, missions with early-delivery bonuses, late-delivery penalties, or high financing costs may find it optimal to minimize trip time. In between are missions with intermediate levels of payment upon delivery and discount rates. Shippers with such missions will find it

[3]For shippers with short trips or a low opportunity cost of waiting for payment, the marginal benefit of quicker delivery is approximately rp.

optimal to use interior solutions which minimize neither trip energy nor trip time.

For example, consider a shipper who wishes to transport lunar ice from the surface of the Moon to satellites orbiting the Earth for use as propellant. Using minimum-energy trajectories, the shipper calculates they can profitably serve satellites as far from the Moon as the geostationary belt (roughly 36,000km above mean sea level on the Earth and 348,000km from the surface of the Moon). The shipper is transporting 200kg of lunar ice, the trip will take around 1 day using an energy-minimizing path, and the relevant discount rate is 1%. The satellite customer will pay $500/kg of lunar ice upon delivery for a total delivery payment of $100,000, and the marginal cost of quicker delivery is $500 per hour of trip time reduced. Is it profitable to spend more money on travel energy in order to reduce delivery time? Checking the conditions in Proposition 1, we find it is: the marginal benefit of quicker delivery is roughly $990 per hour saved, nearly double the marginal cost. The shipper's insistence on minimum-energy transfers is leading them to miss profitable opportunities along costlier paths.

Analysis of the proof of Proposition 1 also reveals how shippers will value improvements in minimum-energy or minimum-time paths when those are optimal. When a minimum-energy path is optimal, the marginal value of a lower-energy orbital transportation solution is the marginal savings per unit of reduced energy consumption minus the marginal present value of receiving payment later per unit of additional energy saved. When a minimum-time path is optimal, the marginal value of a faster orbital transportation solution is the marginal present value of receiving payment sooner minus the marginal cost of energy required to go faster. Sustained improvements in minimum-energy paths, whether through technological improvements or environmental changes, may reduce the marginal cost of quicker delivery by enough that interior paths become optimal.

II. Conclusion

The size of the payment received upon delivery, the shipper's opportunity cost of waiting for payment, and the technological and environmental feasibility of converting money into more energy and less time will determine profit-maximizing orbital transportation solutions. Minimum-energy or minimum-time transfer orbits are unlikely to maximize the shipper's profits when there is a substantial payment upon delivery or a sufficiently high opportunity cost of waiting for payment. Instead, profit-maximizing paths will tend to balance the marginal costs of quicker delivery against the marginal benefits of being paid sooner, producing interior transfer orbits which minimize neither trip energy nor trip time.

Mathematical Appendix

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and minimizes neither trip energy nor trip time when there exist $T > \bar{T}$ and $J > \bar{J}$ such that

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PROOF:

The Lagrangian for the shipper's optimization problem is

$$L = e^{-rT}p - c(J) + \lambda(J - f(T)) + \gamma_J(J - \bar{J}) + \gamma_T(T - \bar{T}).$$

The first-order necessary conditions for an optimum are

$$\begin{aligned} rpe^{-rT} &= -\lambda f'(T) + \gamma_T \\ c'(J) &= \lambda + \gamma_J \\ \lambda(J - f(T)) &= 0 \\ \gamma_J(J - \bar{J}) &= 0 \\ \gamma_T(T - \bar{T}) &= 0. \end{aligned}$$

If the relevant paths between origin and destination are simple (e.g., one- or two-impulse paths) then f will be monotone decreasing and convex, and the first-order conditions will have a unique solution. If the paths are more complex (e.g., multi-impulse paths for satellite proximity operations and deep space missions), there may be multiple solutions to the first-order conditions.

When minimum-energy paths are optimal: In this case we have $J = \bar{J}, T > \bar{T}$ and $\lambda \geq 0, \gamma_T = 0, \gamma_J \geq 0$. The necessary conditions reduce to

$$\begin{aligned} rpe^{-rT} &= -\lambda f'(T) \\ c'(\bar{J}) &= \lambda + \gamma_J. \end{aligned}$$

This implies the shadow price of additional usable energy received for free (λ) is equal to the marginal present value of receiving payment slightly sooner per unit of additional energy used, and the shadow price of a lower-energy orbital transportation solution (γ_J) is the marginal savings per unit of reduced energy consumption minus the marginal present value of receiving payment slightly later per unit of additional energy saved. That is,

$$\lambda = -\frac{rpe^{-rT}}{f'(T)} \geq 0,$$
$$\gamma_J = \underbrace{c'(\bar{J})}_{>0} + \underbrace{\frac{rpe^{-rT}}{f'(T)}}_{\leq 0} \geq 0,$$

with the final inequality holding by assumption that $\gamma_J \geq 0$.

When minimum-time paths are optimal: In this case we have $J > \bar{J}, T = \bar{T}$ and $\lambda \geq 0, \gamma_T \geq 0, \gamma_J = 0$. The necessary conditions reduce to

$$rpe^{-r\bar{T}} = -\lambda f'(\bar{T}) + \gamma_T$$
$$c'(J) = \lambda.$$

This implies the shadow price of additional usable energy received for free is equal to the marginal savings from not having to pay for the energy, and the shadow price of a faster orbital transportation solution (γ_T) is the marginal present value of receiving payment slightly sooner minus the marginal cost of energy required to go slightly faster. That is,

$$\lambda = c'(J) > 0,$$
$$\gamma_T = \underbrace{rpe^{-r\bar{T}}}_{\geq 0} + \underbrace{c'(J)f'(\bar{T})}_{<0} \geq 0,$$

with the final inequality holding by assumption that $\gamma_T \geq 0$.

When interior paths are optimal: In this case we have $J > \bar{J}, T > \bar{T}$ and $\lambda \geq 0, \gamma_T = 0, \gamma_J = 0$. The necessary conditions reduce to

$$rpe^{-r\bar{T}} = -\lambda f'(\bar{T})$$
$$c'(J) = \lambda.$$

As in the minimum-time case, the shadow price of additional usable energy received for free is equal to the marginal savings from not having to pay for the energy. The profit-maximizing orbital transportation solution will have energy use J and delivery time T to satisfy

$$rpe^{-rT} = -c'(J)f'(T).$$